Jun.-Prof. Dr. Caroline Lassueur **FB Mathematik**
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Throughout G denotes finite groups and we fix a splitting p-modular system $(K, \mathcal{O}, k)$ for G with respect to which we define our Brauer characters.

Exercise 45.
Let χ_{reg} denote the (ordinary) regular character of G. Prove that

$$\chi_{\text{reg}} = \sum_{\varphi \in \text{IBr}(G)} \varphi(1)\Phi_\varphi \qquad \text{and} \qquad (\chi_{\text{reg}})^\circ = \sum_{\varphi \in \text{IBr}(G)} \Phi_\varphi(1)\varphi.$$

Exercise 46.
Let N be the smallest normal subgroup of G such that G/N is an abelian p'-group (that is: a group of order not divisible by p). Show that the map

$$\begin{aligned} \Psi : \text{Irr}(G/N) &\to \{\lambda \in \text{IBr}(G) \mid \lambda(1) = 1\} \\ \chi &\mapsto \chi^0 \end{aligned}$$

is a bijection. Conclude that the number of linear Brauer characters of G is $[G : G']_{p'}$, where G' is the commutator subgroup of G.

- Note that $N = G'O^{p'}(G)$, where $O^{p'}(G)$ is the smallest of the normal subgroups $M \trianglelefteq G$ such that G/M is a p'-group
- Recall from that it follows from Corollary 17.2 that every character of an abelian group is linear.

Exercise 47.
Let $\varphi, \lambda \in \text{IBr}(G)$ with λ linear. Prove that $\lambda\varphi \in \text{IBr}(G)$. Moreover, prove that

$$\lambda\Big(\Phi_\varphi\Big)^\circ = \Big(\Phi_{\lambda\varphi}\Big)^\circ.$$