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Let G be a finite group and let $(K, \mathcal{O}, k)$ be a splitting p-modular system for G with respect to which we define our Brauer characters. This exercise connects the character theory of G with the block theory of G.

Definition.
Let $B = \mathcal{O}Gb$ be a block of $\mathcal{O}G$. Then we say that

- $\chi \in \mathrm{Irr}(G)$ **belongs to** B if $e_\chi b \neq 0$, and
- $\varphi \in \mathrm{IBr}(G)$ **belongs to** B if the simple kG-module associated to the irreducible representation affording φ lies in the block $kG\bar{b}$ of kG.

We denote the set of ordinary irreducible characters belonging to B by $\mathrm{Irr}(B)$ and the set of irreducible Brauer characters belonging to B by $\mathrm{IBr}(B)$.

Exercise 48.
Let $\mathcal{O}G = \bigoplus_{i=1}^{r} \mathcal{O}Gb_i$ be the block decomposition $\mathcal{O}G$ for some $r \in \mathbb{N}$, let $B_i = \mathcal{O}Gb_i$ for each $1 \leq i \leq r$ and let $\mathrm{Bl}(G) = \{B_i\}_{i=1}^{r}$.

(a) Show that there exist subsets $X_1, \ldots, X_r$ of $\mathrm{Irr}(G)$ such that for each $1 \leq i \leq r$ we have $KGb_i = \bigoplus_{\chi \in X_i} KGe_\chi$.

(b) Show that $X_i = \mathrm{Irr}(B_i)$ for $1 \leq i \leq r$, and hence

$$\mathrm{Irr}(G) = \bigcup_{B \in \mathrm{Bl}(G)} \mathrm{Irr}(B).$$

(c) Show that

$$\mathrm{IBr}(G) = \bigcup_{B \in \mathrm{Bl}(G)} \mathrm{IBr}(B).$$