Jun.-Prof. Dr. Caroline Lassueur | **FB Mathematik**
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Due date: **Tuesday, 21st of January 2020, 6 p.m.** | WS 2019/20

Exercise 37.
Let F be a field and let A be a finite dimensional F-algebra. Prove the following.

(a) The algebraic closure $\overline{F}$ of F is a splitting field for A.

(b) There is a finite extension $F_1 \mid F$ such that F_1 is a splitting field for A.

Exercise 38.
Let A be a finite dimensional algebra over a commutative ring R. Let V be an A-module and $e \in A$ an idempotent. Prove that

$$\mathrm{Hom}_A(Ae, V) \cong eV$$

as $\mathrm{End}_A(V)$-modules.

Exercise 39.
Let G be a finite group and let K be a field of characteristic 0 that is a splitting field for G. Let $X_1 : G \to \mathrm{GL}_n(K)$ and $X_2 : G \to \mathrm{GL}_n(K)$ be similar matrix representations of G for some $n \geq 1$. Prove that X_1 and X_2 afford the same character.

Exercise 40.
Let G be a finite group. Let $X : G \to \mathrm{GL}_n(\mathbb{C})$ be a complex representation of G of degree $n \geq 1$ and let χ be the character afforded by X. Prove the following.

(a) $\chi(1) = n$.

(b) $\chi(g)$ is a sum of $o(g)$-th roots of unity for all $g \in G$.

(c) $|\chi(g)| \leq \chi(1)$ for all $g \in G$.

(d) $\overline{\chi}$ is also a character of G, defined by $\overline{\chi}(g) = \chi(g^{-1})$ for all $g \in G$.

(e) $\chi(g) = \chi(h^{-1}gh)$ for all $g, h \in G$, i.e. characters are class functions.