Character Theory of Finite Groups — Exercise Sheet 3 TU Kaiserslautern

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Each exercise is worth 4 points.

Exercise 1
Let Q_8 be as defined in Exercise 4, Sheet 2.

(a) Find all the degree 1 representations of Q_8 over $\mathbb{C}$. [Hint: the relations (1) from Ex 4, Sheet 2 may be useful.]

(b) Give the character table of Q_8.

Exercise 2
Let G, H be finite groups with representations $D : G \to \mathrm{GL}_n(\mathbb{C})$, $E : H \to \mathrm{GL}_m(\mathbb{C})$. Show that the map

$$D \otimes E : G \times H \to \mathrm{GL}_{nm}(\mathbb{C}),\ (g,h) \mapsto (D \otimes E)(g,h)$$

is a matrix representation of $G \times H$ with character $\chi_{D\otimes E} = \chi_D \chi_E$ where

$$\chi_{D\otimes E}(g,h) := \chi_D(g)\chi_E(h)$$

for all $(g,h) \in G \times H$.

Exercise 3
Let G, H be finite groups. Show that

$$\mathrm{Irr}_{\mathbb{C}}(G \times H) = \{\chi\psi \mid \chi \in \mathrm{Irr}_{\mathbb{C}}(G), \psi \in \mathrm{Irr}_{\mathbb{C}}(H)\}.$$

Exercise 4
Let G be a finite group. Let $\chi, \lambda \in \mathrm{Irr}_{\mathbb{C}}(G)$ and suppose that λ is linear. Show that $\chi\lambda \in \mathrm{Irr}_{\mathbb{C}}(G)$.