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Throughout G denotes a finite group and K a field of characteristic p. All KG-modules considered are assumed to be finitely generated.

Exercise 29.
Let $H \le G$. Suppose that V is a relatively H-free KG-module with respect to a KH-submodule X, and suppose that W is a relatively H-free KG-module with respect to a KH-submodule Y. Prove that if $X \cong Y$ as KH-modules, then $V \cong W$ as KG-modules.

Exercise 30.
Let $H \le J \le G$. Let U be a KG-module and let V be a KJ-module. Prove the following statements.

(a) If U is H-projective then U is J-projective.

(b) If U is a summand of $V\uparrow_J^G$ and V is H-projective, then U is H-projective.

(c) For any $g \in G$, U is H-projective if and only if ${}^gU$ is ${}^gH$-projective.

Exercise 31.
Let $H \le G$ and $J \le G$. Let U be a KG-module.

(a) Prove that if U is H-projective and W is a KG-module, then $U \otimes_K W$ is H-projective.

(b) Prove that if U is H-projective and W is an indecomposable summand of $U\downarrow_J^G$ then W is $J \cap {}^gH$-projective for some element $g \in G$, and there is a vertex of W that is contained in this subgroup $J \cap {}^gH$.

Exercise 32 (Corrected Jan 6th 2020).
Let U be an indecomposable KG-module with vertex Q ~~and KQ-source S~~ and let L be a subgroup of G containing Q. Prove that there exists an indecomposable direct summand of $U\downarrow_L^G$ with vertex Q.