Representation Theory — Exercise Sheet 8 TU Kaiserslautern

Jun.-Prof. Dr. Caroline Lassueur FB Mathematik

Dr. Niamh Farrell

Due date: **Tuesday, 14th of January 2020, 6 p.m.** WS 2019/20

Throughout G denotes a finite group and K denotes a field of characteristic p. All modules considered are assumed to be finitely generated.

For the first three exercises let Q be a p-subgroup of G and let L be a subgroup of G containing $N_G(Q)$.

Exercise 33 (Lemma 29.2 (a)).
Suppose that V is an indecomposable KL-module with vertex Q and let U be a direct summand of $V{\uparrow}_L^G$ such that V is a direct summand of $U{\downarrow}_L^G$. Then Q is also a vertex of U.

Exercise 34 (Lemma 29.2 (b)).
Suppose that V is an indecomposable KL-module which is Q-projective and there exists an indecomposable direct summand U of $V{\uparrow}_L^G$ with vertex Q. Then V also has vertex Q.

Exercise 35.
An indecomposable module is called a **trivial source module** if it has the trivial module as a source. Let U be an indecomposable KG-module and let V be an indecomposable KL-module. Let f and g be the maps defined in the statement of the Green Correspondence. Prove that if U and V are trivial source modules, then $f(U)$ and $g(V)$ are trivial source modules.

Exercise 36.
Let A be a finitely generated algebra over a commutative ring R. Let P be a projective indecomposable A-module. Prove that there exists an idempotent $e \in A$ such that $P \cong Ae$ as A-modules.