Representation Theory Exercises — Sheet 1

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Due date: **Mo, 05.11.18, 12:00** — WS 18/19

Throughout, R is a commutative ring, A an R-algebra, and all modules are assumed to be *left* modules.

Exercise 1: Show that the A-submodules of the regular A-module are precisely the left ideals of A, and for an ideal $I \lhd A$, A/I is a simple A-module if and only if I is maximal.

Exercise 2 (Homomorphism Theorem for A-modules): Let $f : V \to W$ be an A-homomorphism of A-modules. Show the following:

(a) f is R-linear,

(b) $\ker(f)$ and $\mathrm{Im}(f)$ are A-submodules of V, W respectively,

(c) the map $V/\ker(f) \to \mathrm{Im}(f)$, $v + \ker(f) \mapsto f(v)$, is an isomorphism of A-modules.

Exercise 3: Let V, W be A-modules. Show the following:

(a) $\mathrm{Hom}_A(V, W)$ is an R-module in a natural way,

(b) $\mathrm{End}_A(V)$ is an R-algebra with respect to concatenation of maps.

Exercise 4 (Change of the base ring): Let $\varphi : S \to R$ be a ring homomorphism. Prove that every R-module V can naturally be endowed with the structure of an S-module via the homomorphism φ.