Representation Theory Exercises — Sheet 11

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Due date: **Mo, 04.02.19, 12:00** — WS 18/19

In this exercise sheet, G denotes a finite group, and we refer to a fixed prime p for the construction of Brauer characters. A *p'-group* is a group whose order is not divisible by p.

Exercise 44: Let H be a p'-subgroup of G. Prove that the character Φ_1 is a constituent of the character $(1_H)^G$.

Exercise 45: Let ρ_G denote the regular character of G. Prove that

$$\rho_G = \sum_{\varphi \in \mathrm{IBr}(G)} \varphi(1)\Phi_\varphi \qquad \text{and} \qquad (\rho_G)^0 = \sum_{\varphi \in \mathrm{IBr}(G)} \Phi_\varphi(1)\varphi.$$

Exercise 46: We say that a Brauer character λ of G is *linear* if $\lambda(1) = 1$. Let N be the smallest normal subgroup of G such that G/N is an abelian p'-group. Show that the map

$$\Psi : \mathrm{Irr}(G/N) \to \{\lambda \in \mathrm{IBr}(G) \mid \lambda(1) = 1\}$$
$$\chi \mapsto \chi^0$$

is a bijection. Conclude that the number of linear Brauer characters of G is $[G : G']_{p'}$.
[Note that $N = G'O^{p'}(G)$, where $O^{p'}(G)$ is the smallest of the normal subgroups $M \trianglelefteq G$ such that G/M is a p'-group.]

Exercise 47: Let $\varphi, \lambda \in \mathrm{IBr}(G)$ with λ linear. Prove that $\lambda\varphi \in \mathrm{IBr}(G)$. Moreover, prove that $\lambda(\Phi_\varphi)^0 = (\Phi_{\lambda\varphi})^0$.