Representation Theory Exercises — Sheet 3

Prof. Dr. G. Malle — Dr. N. Farrell

Due date: **Monday, 19.11.18, 12:00** — WS 18/19

Unless otherwise stated, R is a commutative ring, A is an R-algebra, and all modules are left modules.

Exercise 8: Let U, V_1, V_2 be A-modules, each of which has a composition series. Prove that if $U \oplus V_1 \cong U \oplus V_2$ then $V_1 \cong V_2$.

Exercise 9: Let V be an A-module which has a composition series. Prove that V admits a direct sum decomposition

$$V = V_1 \oplus \cdots \oplus V_n \quad (n \in \mathbb{N})$$

into indecomposable A-submodules which are uniquely determined up to isomorphism and up to order of occurrence.

Exercise 10: Let A be an R-algebra.

(a) Let V be a completely reducible A-module, and let $U \leq V$ be an A-submodule of V. Prove that both U and V/U are completely reducible.

(b) Suppose that A is a semisimple R-algebra, and let B be an ideal of A. Prove that the R-algebra A/B is semisimple.

Exercise 11: Let K be a field and let A be a finite-dimensional K-algebra.

(a) Let $z \in Z(A)$ be a nilpotent element. Prove that $z \in J(A)$.

(b) Let V be an A-module. Prove that V is completely reducible if and only if $J(A)V = \{0\}$.

(c) Prove that if V is a completely reducible A-module, then $\mathrm{End}_A(V)$ is a semisimple K-algebra.