Representation Theory Exercises — Sheet 8

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Due date: **Mo, 14.01.19, 12:00** | WS 18/19

Please note: the next exercise class is on 10.01.19 (no class on 03.01.19)

Fröhliche Weihnachten!

In this exercise sheet, K denotes a field and G denotes a finite group.

Exercise 29: Prove that if G has odd order, then for every $g \in G$ we have

$$\sum_{\chi \in \mathrm{Irr}(G)} \nu(\chi)\chi(g) = 1.$$

Exercise 30: Let $G = D_{2n}$ be a dihedral group of order $2n$, $n \geq 3$. Prove that $\nu(\chi) = 1$ for all irreducible characters χ of G.

[Hint: calculate the number of elements $g \in G$ such that $g^2 = 1$]

Exercise 31: Let V be a relatively H-free KG-module with respect to the KH-submodule X, and let W be a relatively H-free KG-module with respect to the KH-submodule Y. Prove that if $X \cong Y$ as KH-modules, then $V \cong W$ as KG-modules.

Exercise 32: Let $H \leq G$, X be a KH-module and V be a KG-module. Prove that the map $\Phi : \mathrm{Hom}_{KH}(V|_H, X) \to \mathrm{Hom}_{KG}(V, X^G)$ defined by

$$\Phi(\gamma)(u) := \sum_{s \in S} s \otimes \gamma(s^{-1}v) \quad \text{for } \gamma \in \mathrm{Hom}_{KH}(V|_H, X),\ v \in V,$$

is an isomorphism.

Exercise 33: Let A be a (finite dimensional) algebra. Prove that if

$$0 \longrightarrow M_1 \longrightarrow M \longrightarrow M_2 \longrightarrow 0$$

is a short exact sequence of A-modules, then so is

$$0 \longrightarrow \mathrm{Hom}_A(P, M_1) \longrightarrow \mathrm{Hom}_A(P, M) \longrightarrow \mathrm{Hom}_A(P, M_2) \longrightarrow 0$$

for every projective A-module P.

Exercise 34: Let $H \leq G$. Prove that a sequence

$$0 \longrightarrow X_1 \xrightarrow{\alpha} X \xrightarrow{\beta} X_2 \longrightarrow 0$$

of KH-modules is exact if and only if the induced sequence

$$0 \longrightarrow X_1^G \xrightarrow{\alpha^G} X^G \xrightarrow{\beta^G} X_2^G \longrightarrow 0$$

of KG-modules is exact.